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BCA (Sem-I) – Math

(103) GE-I

2022

Time : 3 hours

Full Marks : 40

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer ***all*** *sections as directed.*

Section–A

(Compulsory)

1. Pick up the correct alternative for each of the following questions : 1×10=10

(a) If $y = e^{mx}$, then y_n is equal to :

(i) m

(ii) n

(iii) $m^n e^{mx}$

(iv) $n^m e^{nx}$

(b) If $y = \sin(ax+b)$ then y_n is equal to :

(i) $\sin(ax+b)$

(ii) $a^n \sin\left(ax+b+\frac{n\pi}{2}\right)$

(iii) $a^n \sin(ax+b+n\pi)$

(iv) None of these

(c) The maximum value of $\cos x$ in R is :

1π
1×180

(i) 0

(ii) -1

(iii) 1

(iv) None of these

(d) If u be a function of x and y then

$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$ is equal to :

(i) du

(ii) df

(iii) ∂u

(iv) None of these

(e) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ is equal to :

(i) $\frac{\pi}{2}$

(ii) $\frac{\pi}{4}$

(iii) π

(iv) None of these

(f) The area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

(i) $\pi^2 a^2 b$

(ii) πab

(iii) a^2b

(iv) None of these

(g) $\int_0^\infty \frac{1}{1+x^2}dx$ is equal to :

$(\tan^{-1}x)_0^\infty$

$\tan^{-1}\infty - \tan^{-1}0$

$= \frac{\pi}{2} - 0$

(i) 0

(ii) ∞

(iii) $\frac{\pi}{2}$

(iv) None of these

(h) The number of arbitrary constants in the general solution of linear different equation of second order is/are :

(i) 0 (ii) 1

(iii) 2 (iv) 3

(i) Integrating factor of the differential equation $\frac{dy}{dx} + y\cot x = 2\cos x$ is :

(i) $\sin x$

(ii) $\cos x$

(iii) $\sec x$

(iv) $\tan x$

(j) The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is :

(i) $\log y = kx$

(ii) $ny = k$

(iii) $y = kx$

(iv) $y = k \log x$

Section–B

(Short answer type questions)

Answer any **three** questions of the following :

5×3=15

2. State and prove Leibnitz theorem.

3. State and prove Maclaurin theorem.

4. Find the maximum and minimum value of

$3x^3 - 7x^2 + 15x - 10$.

5. Explain the method of finding the solution of linear differential equation of first order.

6. Explain method of finding the complementary function (CF) of a differential equation with constant coefficients.

7. Prove that

$$\int_0^{2a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

Section–C

(Long answer type questions)

Answer any **three** questions of the following :

5×3=15

8. If $y = e^{a\sin^{-1}x}$, prove that

$$(1-x^2)y_2 - xy_1 - a^2 y = 0,$$

9. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

10. Evaluate $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\sin^2 x + b^2\cos^2 x} dx$ or

$$\int_0^{\frac{\pi}{2}} \log \sin x \, dx$$

11. Apply Maclaurin's theorem to prove that

$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots\ldots + (-1)^n \frac{x^n}{n}$$

12. Solve any **one** of the following :

(i) $\frac{dy}{dx} + 1 = e^{x-y}$

(ii) $\cos^2 x - \frac{dy}{dx} + y = \tan x$

13. Solve any **one** of the following :

(i) $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 4y = x^4$

(ii) $\frac{d^2y}{dx^2} + y = \sin 2x$

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