BCA (Sem-I) – Math (103)

GE-I

2023

Time : 1.30 hours

Full Marks : 40

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

*Answer **all** sections as directed.*

Section–A

Objective type Questions

Compulsory

1. Choose the correct alternative for each of the following questions : 1×10=10

   (a) If $y = x^n$ then y_{n+1} is equal to :

   (i) n

(ii) $\lfloor n$

(iii) 0

(iv) None of these

(b) If $y = e^{ax}$ then y_n is equal to :

(i) $a^n e^{ax}$

(ii) ae^{ax}

(iii) e^{ax}

(iv) None of these

(c) The maximum value of sin x in R is :

(i) 0

(ii) 1

(iii) −1

(iv) None of these

(d) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{cos x}}{\sqrt{sin x} + \sqrt{cos x}} dx$ is equal to :

(i) $\frac{\pi}{2}$

(ii) $\frac{\pi}{4}$

(iii) π

(iv) None of these

(e) $\int_0^{\frac{\pi}{2}} sin x dx$ is equal to :

(i) 1

(ii) −1

(iii) 1/2

(iv) None of these

(f) The area of the circle $x^2 + y^2 = 16$ is :

(i) 4

(ii) 16π

(iii) $4\pi^2$

(iv) None of these

(g) The number of arbitrary consants in the general solution of differential equation of fourth order is/are :

(i) 1

(ii) 2

(iii) 3

(iv) 4

(h) Integrating factor of the differential equation $\frac{dy}{dx}+2xy=3$ is :

(i) x^2

(ii) $\frac{x^2}{2}$

(iii) $2x^2$

(iv) None of these

(i) The general solution of the differential equation $\frac{dy}{dx}=\frac{x}{y}$ is :

(i) $\log y = kx$

(ii) $y = kx$

(iii) $y = k\log x$

(iv) None of these

(j) The order and degree of the differential equation $\frac{d^2y}{dx^2}+6\left(\frac{dy}{dx}\right)^3+5y=\sin x$ respectively :

(i) 2,3

(ii) 3,2

(iii) 2,1

(iv) None of these

Section–B

Short Answer type Questions

Answer any **three** questions of the following :

5×3=15

2. State and prove Euler's theorem on homogeneous function.

3. State and prove Taylor's theorem with Lagrange's form of remainder.

4. Find the maximum and minimum value of $2x^3-21x^2+36x-20$.

5. Explain the method of finding the solution of homogeneous differential equation.

6. Explain the method of finding expression of $\cos x$ in Maclaurine's series.

7. Prove that $\int_0^a f(x)dx=\int_0^a f(a-x)dx$.

Section–C

Long Answer type Questions

Answer any **three** questions of the following :

5×3=15

8. If $y=\sin(m\sin^{-1}x)$, then prove that $(1-x^2)y_2-xy_1+m^2y=0$

9. If $u=\sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\tan u$$

$u.v = \frac{du}{dx}.v+\frac{dv}{dx}.u$

10. Evaluate $\int_0^{\frac{\pi}{2}}\frac{dx}{a^2\cos^2x+b^2\sin^2x}$ or $\int_0^{\frac{\pi}{2}}\log\cos x\,dx$

11. Apply Maclaurine's theorem to prove that

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+....+\frac{x^n}{n!}+....$$

12. Solve any **one** of the following :

(a) $\frac{dy}{dx}+\tan x.y=\sec x$

(b) $xdy-ydx-\sqrt{x^2-y^2}dx=0$

13. Solve any **one** of the following :

(a) $\frac{d^2y}{dx^2}+a^2y=x\cos x$

(b) $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=x^3e^{x2}$

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