

2023**(Session : 2021-24)***Time : 1½ hours**Full Marks : 40*

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the sections as directed.

Section – A

Answer any **four** questions of the following :

4×6 = 24

1. (a) State Archimedian property of \mathbb{R} .
- (b) Find the infimum and supremum of the following sets:
 - (i) $\{0, 1, 2, 5, 8, 10\}$
 - (ii) $\{2 + \frac{1}{n} : n \in \mathbb{N}\}$

2. Show that limit of a sequence, if it exists, is unique.
3. Define sequence. Prove that a monotonically increasing sequence which is bounded above is convergent.
4. State and prove Ratio test.
5. State and prove Cauchy's general principle of convergence for series.
6. Define absolute convergence and conditional convergence with examples.

Section – B

Answer any **four** questions of the following :

4×4 = 16

7. Show that the sequence $\left\{1 + \frac{1}{n}\right\}$ converges to limit 1.
8. Show that the sequence

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n} \text{ is convergent.}$$

9. Show that $\lim_{n \rightarrow \infty} n^{\frac{1}{n}} = 1$.

10. Show that the series

$1 - \frac{1}{2} + \frac{1}{2^2} - \frac{1}{2^3} + \frac{1}{2^4} - \dots$ is absolutely convergent.

11. Give examples of monotonic increasing and decreasing sequences.

12. Give examples of finite and infinite sets.

