

2024

Time : $1\frac{1}{2}$ hours

Full Marks : 40

Candidates are required to give their answers in their own words as far as practicable.

The figures in the margin indicate full marks.

Answer from both the sections as directed.

Section-A

1. Answer any four questions of the following :

6×4=24

1. (a) (i) State order and completeness property of \mathbb{R} .

(ii) Find the infimum of the sets (a, b) and

$$\left\{ 3 + \frac{1}{n^2} : n \in \mathbb{N} \right\}.$$

(iii) Find the supremum of the sets $[2,3]$

$$\text{and } \left\{ 1 + \frac{1}{n^2} : n \in \mathbb{N} \right\}$$

(b) State and prove Cauchy's general principle of convergence.

(c) Define convergent sequence. Prove that limit of a sequence, if it exists, is unique.

(d) State and prove Root test.

(e) Prove that every absolutely convergent series is convergent but the converse need not be true.

(f) State and prove Leibniz's test.

Section-B

Answer any four questions of the following :

$$4 \times 4 = 16$$

2. (a) Show that the sequence $\left\{ 1 + \frac{1}{n} \right\}$ converges to the limit 1.

(b) Using Cauchy's general principle of convergence, show that the sequence $\{a_n\}$ where $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$ is not convergent.

(c) Test the convergence of the series $1 + \frac{1}{2^2} + \frac{1}{3^3} + \frac{1}{4^4} + \dots + \frac{1}{n^n} + \dots \infty$

(d) Show that the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots \infty$ is conditionally convergent.

(e) Give examples of countable and uncountable sets.

(f) Give examples of convergent, divergent and oscillatory sequences and alternating series.

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